DET KGL. DANSKE VIDENSKABERNES SELSKAB
MATEMATISK-FYSISKE MEDDELELSER, BIND XXIV, NR. 7

A SPECIAL CASE OF VELOCITY FOCUSING IN A MAGNETIC DEFLECTING FIELD

BY

JØRGEN KOCH

KØBENHAVN
I KOMMISSION HOS EJNAR MUNKSGAARD
1948

CONTENTS

Printed in Denmark
Bianco Lunos Bogtrykkeri

I. Introduction.

The production of ion beams of high degree of homogeneity with respect to mass and energy is of increasing importance for a number of experimental investigations. In connection with experiments made with the mass spectrograph of the Institute for Theoretical Physics of the University of Copenhagen[1, 2] a very simple method has been worked out by which it is possible to meet rather strict demands for homogeneity, even though the d. c. high voltage used for accelerating the ions is not very constant[3]. This is of particular importance when very high voltages are used, for in this case a smoothing of the voltage ripple and other fluctuations may be inconvenient or impossible to carry out satisfactorily.

An essential feature of the proposed method is the application of a particular form of velocity focusing which even under the conditions mentioned makes it possible to focus the ions on a collecting cylinder or target placed behind the wedge-shaped magnetic field that is used to resolve the beam. To indicate the experimental conditions under which this method was applied[3] it should be pointed out that the mass spectrograph used here works with a very narrow beam produced by ions from a low voltage ion source. This beam is accelerated and collimated by means of a system of electrostatic lenses which makes use of high voltages. The relative energy spread of the ions is very slight, since the ions leave the ion source with very small energies. The magnetic deflecting field is kept constant by means of a large storage battery.

In the following a detailed account will be given of the general principles of the method of focusing, which has so far been only briefly described (loc. cit.). The treatment of the problem by means of geometrical optics shows that a focusing can be carried out for all kinds of charged particles traversing an arbitrarily extended homogeneous wedge-shaped magnetic field. The calculations show certain features of the passage of charged particles through magnetic fields which may be useful in the construction of types of apparatus other than the one mentioned here.

II. General Description of the Method of Focusing.

Let us consider a narrow beam of ions of uniform mass and energy coming from the ion source I (fig. 1) and moving in parallel paths towards R, where the beam enters a homogeneous wedge-shaped magnetic field AOB with central angle ϑ. In the magnetic field the ions will be deflected in circular paths around the apex O with a radius

$$r = H^{-1}\sqrt{2\,VM/e}, \tag{1}$$

where V is the accelerating potential, e/M is the specific charge of the particles, and H is the magnetic field strength. At R' the ions continue their paths along a straight line.

Now, if the accelerating potential V is not constant, but changes as a function of time, so that $\Delta V = \Delta V(t)$, the radius of curvature of the ion paths will change correspondingly and the ion beam will strike the plane $n' - n''$ at various places. On a fluorescent screen placed there, slow variations of ΔV may be observed as oscillations about a central position, while rapid variations manifest themselves in an increase of the natural line breadth. If the beam contains several ion species differing slightly in mass, the beam at $n' - n''$ will be resolved into several components lying close together, provided that both the inherent line breadth and the oscillations of the beams are small compared with the mass dispersion.

To compensate for these oscillations it is possible, as will be proved below, to let the ion beam pass between the electrodes of a parallel plate condenser, placed at G, one plate of which (a) is grounded while the other (b) is supplied with a voltage E which is proportional to the fluctuations ΔV of the acceleration potential from its average value:

$$E = E_0 \Delta V, \quad \Delta V \ll V. \tag{2}$$

The proportionality factor E_0 takes account of the extent to

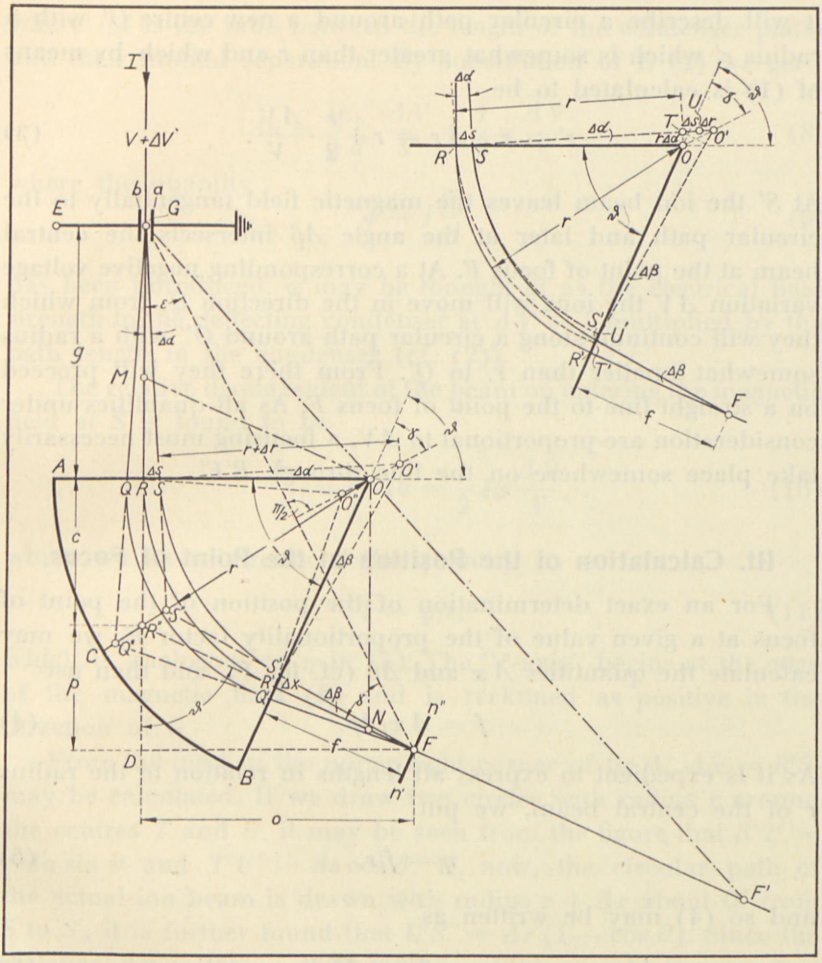


Fig. 1. Graphical construction for the computation of the point of focus F indicating the characteristic features of the construction of the ion paths by means of geometrical optics as discussed in section V.

which the deflecting voltage on the condenser must depend on the voltage variations ΔV. To show qualitatively that the intended focusing is obtained by this means, let us first assume that ΔV is positive. The ion beam then will suffer a slight angular deflection $\Delta \alpha = \text{const.} \times \Delta V$ in the direction of the plate a, so that it will appear to have been emitted from a virtual source G in the middle of the condenser. Entering the magnetic field at S

it will describe a circular path around a new centre O' with a radius r' which is somewhat greater than r and which by means of (1) is calculated to be

$$r' = r + \Delta r = r + \frac{r}{2} \cdot \frac{\Delta V}{V}. \tag{3}$$

At S' the ion beam leaves the magnetic field tangentially to the circular path and later at the angle $\Delta\beta$ intersects the central beam at the point of focus F. At a corresponding negative voltage variation ΔV the ions will move in the direction Q, from which they will continue along a circular path around O'' with a radius somewhat smaller than r, to Q'. From there they will proceed on a straight line to the point of focus F. As all quantities under consideration are proportional to ΔV, a focusing must necessarily take place somewhere on the line through $R'F'$.

III. Calculation of the Position of the Point of Focus.

For an exact determination of the position of the point of focus at a given value of the proportionality factor E_0 we may calculate the quantities Δx and $\Delta\beta$ (cf. fig. 1), and then use

$$f = \Delta x/\Delta\beta. \tag{4}$$

As it is expedient to express all lengths in relation to the radius r of the central beam, we put

$$\eta = f/r, \tag{5}$$

and so (4) may be written as

$$\eta = \Delta x/r\,\Delta\beta. \tag{6}$$

The "η-axis" introduced here begins at R' and is reckoned as positive in the direction of F.

For the calculation of (6) we shall first determine the angular deflection in the plate condenser, the field of which is assumed to be homogeneous*. In the case of small deflections we obtain the well-known expression

$$\Delta\alpha = \frac{lE}{2\,dV}, \quad (E \ll V), \tag{7}$$

* The effects of stray fields are not considered and only first order terms are retained in the calculations.

where l/d is the ratio between the length of the condenser plates and their mutual separation. By substitution of E (2) we get

$$\Delta a = \frac{lE_0}{2\,d} \cdot \frac{\Delta V}{V} = \frac{1}{2}\varphi\frac{\Delta V}{V}, \tag{8}$$

where the quantity

$$\varphi = l\frac{E_0}{d} \tag{9}$$

has been introduced. φ may be thought of as the electrical field strength in the deflecting condenser at $\Delta V = 1$ multiplied by the path length in the condenser (cf. (2)).

The relative displacement of the beam on entering the magnetic field at S is found to be

$$\frac{\Delta s}{r} = \frac{g}{r}\Delta a = \frac{1}{2}\xi\varphi\frac{\Delta V}{V}, \tag{10}$$

where we have introduced the quantity

$$\xi = g/r, \tag{11}$$

which is analogous to η in (5). The "ξ-axis" begins at the edge of the magnetic field (R) and is reckoned as positive in the direction of G.

From the inset in the upper right corner of fig. 1, $\Delta x = R'S'$ may be calculated. If we draw two circles with radius r around the centres T and U, it may be seen from the figure that $R'T' = r\,\Delta a \sin\vartheta$ and $T'U' = \Delta s \cos\vartheta$. If, now, the circular path of the actual ion beam is drawn with radius $r + \Delta r$ about O' from S to S', it is further found that $U'S' = \Delta r\,(1 - \cos\vartheta)$. Since the distance $\Delta x = R'S' = R'T' + T'U' - U'S'$, we obtain

$$\frac{\Delta x}{r} = \Delta a \sin\vartheta + \frac{\Delta s}{r}\cos\vartheta - \frac{\Delta r}{r}(1 - \cos\vartheta). \tag{12}$$

Substituting Δa (8), $\Delta s/r$ (10), and $\Delta r/r$ (3), we obtain

$$\frac{\Delta x}{r} = \frac{1}{2}(\varphi\sin\vartheta + \xi\varphi\cos\vartheta + \cos\vartheta - 1)\frac{\Delta V}{V}. \tag{13}$$

From the inset in fig. 1 it is found correspondingly that

$$\Delta\beta = [\sin\vartheta\,(\Delta s + \Delta r) - r\,\Delta a\cos\vartheta]/r, \tag{14}$$

from which, with the same substitutions as used above, we obtain

$$\Delta\beta = \frac{1}{2}(\xi\varphi \sin\vartheta + \sin\vartheta - \varphi\cos\vartheta)\frac{\Delta V}{V}. \tag{15}$$

Substituting (13) and (15) in (6), we finally find an expression for the relative position of the point of focus

$$\eta = \frac{\varphi\sin\vartheta + \xi\varphi\cos\vartheta + \cos\vartheta - 1}{\xi\varphi\sin\vartheta + \sin\vartheta - \varphi\cos\vartheta}. \tag{16}$$

From this it appears that a focusing on the positive η-axis may take place at suitable values of φ, ξ and ϑ.

IV. The Proportionality Factor E_0.

In order to apply this method in practice we must calculate $E_0 = \varphi \cdot d/l$ for the values of η, ξ and ϑ given by the experimental conditions. For the discussion of the conditions of focusing we therefore solve (16) for φ, obtaining

$$\varphi = \frac{1 + \eta\sin\vartheta - \cos\vartheta}{(1-\eta\xi)\sin\vartheta + (\eta + \xi)\cos\vartheta}. \tag{17}$$

It should be remarked here that if, for the calculation of φ, the relative distances from F and G, respectively, to the intersection R'' of the η- and ξ-axes are used instead of η and ξ, viz.

$$\left.\begin{aligned} \eta' &= f/r + c/r = \eta + \operatorname{tg}\vartheta/2 \text{ and} \\ \xi' &= g/r + c/r = \xi + \operatorname{tg}\vartheta/2, \end{aligned}\right\} \tag{18}$$

we obtain

$$\varphi = \frac{\eta'\sin\vartheta}{\eta' + \xi' - \eta'\xi'\sin\vartheta}. \tag{19}$$

The reason why this expression is of a somewhat simpler form than (17) is that the bisector OC may be viewed as the principal plane of the cylindrical lens, since beams leading from G to F are apparently deflected there once (cf. section V, 1). In what follows the expression (17) for φ will, however, be used, as this offers the best possibility for carrying through a discussion of the conditions of focusing.

A general mathematical analysis of the function φ (17) gives the following result. If we present φ as a function of η with ξ

as parameter at a fixed value of ϑ, we get a set of equilateral hyperbolas which are symmetrical with respect to a pair of axes rotated by $\pi/4$ from the η, φ-system and having their centres at the points

$$\eta_c = \frac{\xi \cos\vartheta + \sin\vartheta}{\xi \sin\vartheta - \cos\vartheta}; \quad \varphi_c = -\frac{\sin\vartheta}{\xi \sin\vartheta - \cos\vartheta}. \tag{20}$$

The centres of the hyperbolas for various values of ξ lie on the straight line

$$\varphi_c(\eta_c) = -\sin^2\vartheta\,(\eta_c - \operatorname{ctg}\vartheta). \tag{21}$$

It is further characteristic that the hyperbolas for all values of ξ pass through the two points X and Y with coordinates

$$\eta_X = \operatorname{ctg}\vartheta; \quad \varphi_X = \sin\vartheta, \tag{22}$$

$$\eta_Y = (\cos\vartheta - 1)/\sin\vartheta; \quad \varphi_Y = 0. \tag{23}$$

A degenerate case arises when $\xi = \operatorname{ctg}\vartheta$. The function $\varphi(\eta)$ is then of the form

$$\varphi(\eta) = \sin^2\vartheta \left(\eta - \frac{\cos\vartheta - 1}{\sin\vartheta}\right), \tag{24}$$

i. e. φ is linearly dependent on η. Finally it should be stated that the function $\varphi(\eta)$ has a singularity ($\varphi = \pm\infty$) when the denominator in formula (17) vanishes, i. e. when

$$(1 - \eta\xi)\sin\vartheta + (\eta + \xi)\cos\vartheta = 0. \tag{25}$$

In connection with the physical problem discussed here only the course of $\varphi(\eta)$ at positive values of η and ξ is of interest. The reader is referred to section VI, where $\varphi(\eta)$ is represented graphically for $\vartheta = \pi/3$, $\pi/2$, π and $3\pi/2$. The connection between the above mentioned properties of the function $\varphi(\eta)$ ((22)—(25)) and the geometrical treatment of the problem of focusing will be discussed in detail in section V.

V. Characteristic Features of the Construction of the Ion Paths by Means of Geometrical Optics.

In what follows attention will be called to some characteristic features of the construction of the ion paths by means of geometrical optics which are convenient to use in the graphical treatment of the focusing method mentioned here and are furthermore of particular importance in its practical application.

(1) As appears from fig. 1, the η and ξ-axes intersect at the point R'' on the bisector OC of the wedge-shaped field so that the deflection of the central beam is seen to be ϑ. As mentioned above (p. 8), the bisector OC, however, may generally be considered as a principal plane of the prism, all ion beams here apparently being subjected to one single deflection. This result appears by putting

$$(g + c)\,\Delta\alpha = (f + c)\,\Delta\beta, \tag{26}$$

from which follows

$$(\xi + \operatorname{tg} \vartheta/2)\,\Delta\alpha = (\eta + \operatorname{tg} \vartheta/2)\,\Delta\beta. \tag{27}$$

Substituting here (8) and (15), and replacing $\vartheta/2$ by ϑ we obtain an expression for φ that is in accordance with the expression (17). The centres O' and O'' of the circular paths SS' and QQ' are found by making the perpendiculars to the beams at the points S, S' and Q, Q' intersect.

(2) It is now to be demonstrated that (a) the centres O, O' and O'' of the circular arcs RR', SS' and QQ', respectively, lie on a straight line, and (b) the line through O, O', and O'' is perpendicular to the connecting line GF.

(a) From the inset in the upper right corner of fig. 1 it is found that

$$\operatorname{tg}(\vartheta - \gamma) = \frac{r\,\Delta\alpha}{\Delta s + \Delta r} \tag{28}$$

which, after substitution of $\Delta\alpha$ (8), Δs (10) and Δr (3) may be transformed into

$$\operatorname{tg}(\vartheta - \gamma) = \frac{\varphi}{\xi\varphi + 1}. \tag{29}$$

As this expression is independent of ΔV the centres O, O', and O'' must lie on a straight line.

(b) On the basis of the above assertion, we have $\sphericalangle\, GFR'' = \gamma$ and consequently $\varepsilon = \sphericalangle\, R''GF = \vartheta - \gamma$. From the right triangle

FDG we get

$$\operatorname{tg} \varepsilon = \frac{o}{g + c + p}. \tag{30}$$

Substituting here

$$\left.\begin{aligned} o/r &= (\eta + \operatorname{tg} \vartheta/2) \sin \vartheta, \\ p/r &= (\eta + \operatorname{tg} \vartheta/2) \cos \vartheta, \\ c/r &= \operatorname{tg} \vartheta/2 \end{aligned}\right\} \tag{31}$$

and then putting (29) = (30), we get

$$\frac{\varphi}{\xi \varphi + 1} = \frac{\sin \vartheta\, (\eta + \operatorname{tg} \vartheta/2)}{\xi + \operatorname{tg} \vartheta/2 + \cos \vartheta\, (\eta + \operatorname{tg} \vartheta/2)}, \tag{32}$$

which again leads to an expression for φ corresponding to (17). Thus it is proved that the straight line through the centres O, O' and O'' is perpendicular to the line GF. The application of this fact is particularly suitable for use in the geometrical construction of the ion paths.

(3) A focusing at the point F' (cf. fig. 1), which is lying on the straight line through the point of divergence G and the apex O cannot take place. This means

$$\operatorname{tg} \varepsilon = r/g = 1/\xi. \tag{33}$$

Using the formulas (30) and (31), this leads to

$$\cos \vartheta\, (\eta + \operatorname{tg} \vartheta/2) + \operatorname{tg} \vartheta/2 + \xi - \xi \sin \vartheta\, (\eta + \operatorname{tg} \vartheta/2) = 0. \tag{34}$$

By the substitution of ϑ for $\vartheta/2$ (34) may be transformed to

$$(1 + \cos \vartheta)\,(\sin \vartheta - \eta \xi \sin \vartheta + \eta \cos \vartheta + \xi \cos \vartheta) = 0. \tag{35}$$

This formula can only be satisfied for all values of ϑ if the expression in the parenthesis on the right is equal to zero, which, however, according to (25) corresponds to $\varphi = \pm \infty$. Hence we cannot obtain a focusing at the point F' or its immediate proximity, as the voltage variations of the deflecting condenser must be small compared with the acceleration potential (cf. (7)). We shall discuss in section VII, in connection with the graphical representations of φ in figs. 3—6, how close it is practically possible to approach the point F'.

Finally, attention should be called to the fact that the point F' is also of particular importance in the general theory of the passage of ion beams through a wedge-shaped magnetic field. R. Herzog[4], N. F. Barber[5] and W. E. Stephens[6] have independently shown that a slightly divergent bundle of charged particles with uniform energy coming from G will be focused at F' (direction focusing). It is generally said that F' is the image of G and the formula (25) gives the conditions that the image be formed.

(4) If the deflecting condenser (G) is placed at M, the intersection of the ξ-axis and the perpendicular to BO at O, then φ is linearly dependent on η. For under the conditions mentioned we have $\sphericalangle RMO = \vartheta$ and $\xi = g/r = \operatorname{ctg} \vartheta$ and the formula for $\varphi(\eta)$ then has the form (24) in accordance with the above insertion.

(5) If the beams are to be focused at N, the intersection of the η-axis and the perpendicular to AO at O, the deflecting condenser may be placed anywhere on the ξ-axis, and φ will have the same value for all positions. For under the conditions mentioned we have $\sphericalangle ONR' = \vartheta$ and $\eta = f/r = \operatorname{ctg} \vartheta$, which according to (22) corresponds to $\varphi = \sin \vartheta$ for all values of ξ.

It should be added that the point N is further characterised by being the one focus (in the ordinary optical sense) of the magnetic prism, since a parallel bundle of charged particles coming from I in the direction of R will just be focused here[4, 5, 6]. This form of focusing is closely connected with the peculiarity mentioned just above, which perhaps appears most clearly from a drawing of the ion paths at various values of ξ.*

(6) The derivation of the formula (17) for φ and the detailed discussion of the conditions of focusing have so far been carried out in connection with fig. 1, where an angle of deflection $\vartheta < \pi/2$ is chosen. By means of the above subsections (1)–(5) it is, however, easy to evaluate conditions in the case of an arbitrary angle.

Thus in fig. 2 an angle ϑ which is between π and $3\,\pi/2$ has been chosen as an example. The general features of the geometrical construction are easily recognized; in the case of $\vartheta > \pi$ there

* Instead of characterizing the position of the focus N by $\eta = \operatorname{ctg} \vartheta$ one often, as mentioned on p. 8, uses its relative distance from the principal plane OC of the prism, which is easily calculated to be $\eta' = 1/\sin \vartheta$. – The fact that the point M is the second focus of the prism is closely connected with the consideration mentioned in subsection (4) above.

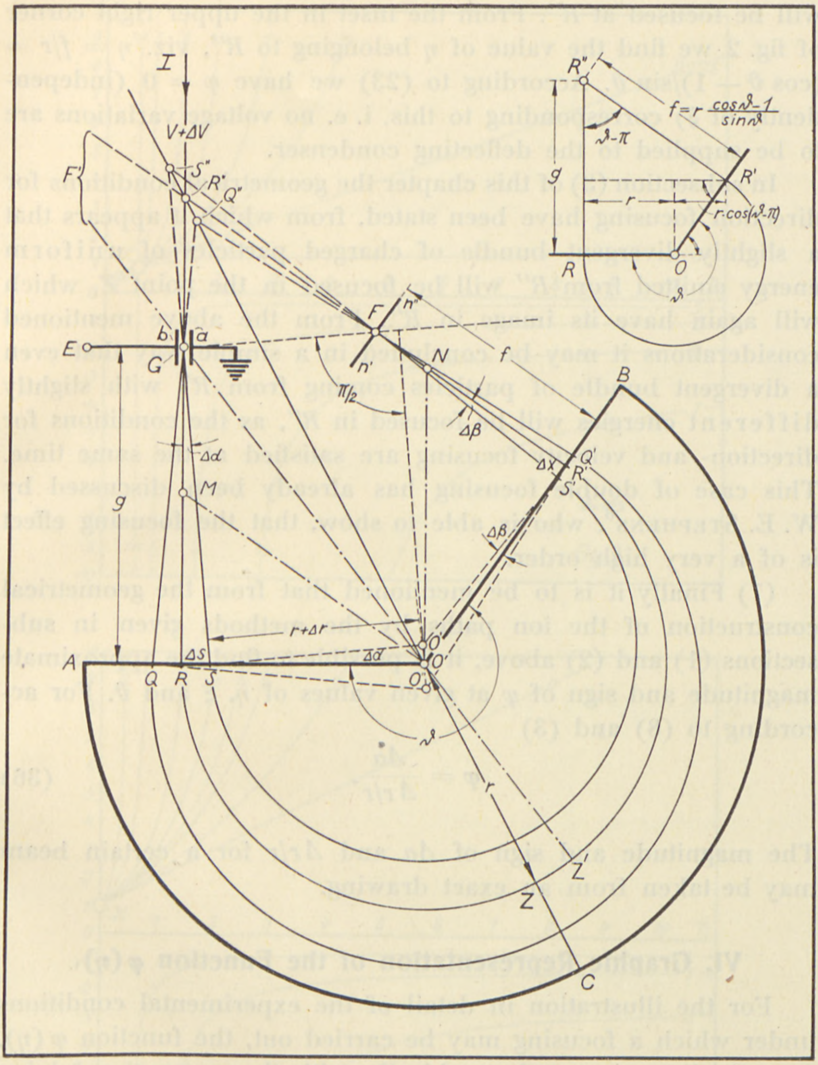


Fig. 2. Graphical construction of the point of focus at a central angle ϑ between π and $3\,\pi/2$.

is, however, an additional feature which should be mentioned. If the point of focus F coincides with R'', which is the intersection of the η- and ξ-axes, the magnetic field will itself have a velocity focusing effect, i. e. an ion beam coming from I with the energies $V + \Delta V(t)$ without any influence from a deflecting condenser will be focused at R''. From the inset in the upper right corner of fig. 2 we find the value of η belonging to R'', viz. $\eta = f/r = (\cos\vartheta - 1)/\sin\vartheta$. According to (23) we have $\varphi = 0$ (independently of ξ) corresponding to this, i. e. no voltage variations are to be supplied to the deflecting condenser.

In subsection (3) of this chapter the geometrical conditions for direction focusing have been stated, from which it appears that a slightly divergent bundle of charged particles of uniform energy emitted from R'' will be focused in the point Z, which will again have its image in R''. From the above mentioned considerations it may be concluded in a simple way that even a divergent bundle of particles coming from R'' with slightly different energies will be focused in R'', as the conditions for direction- and velocity focusing are satisfied at the same time. This case of double focusing has already been discussed by W. E. Stephens[6], who is able to show, that the focusing effect is of a very high order.

(7) Finally it is to be mentioned that from the geometrical construction of the ion paths by the methods given in subsections (1) and (2) above, it is possible to find the approximate magnitude and sign of φ at given values of η, ξ and ϑ. For according to (8) and (3)

$$\varphi = \frac{\Delta a}{\Delta r/r}. \tag{36}$$

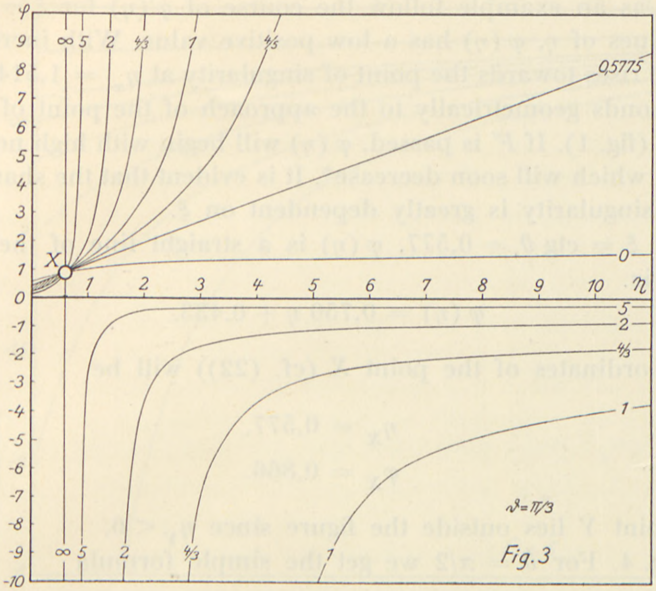
The magnitude and sign of Δa and $\Delta r/r$ for a certain beam may be taken from an exact drawing.

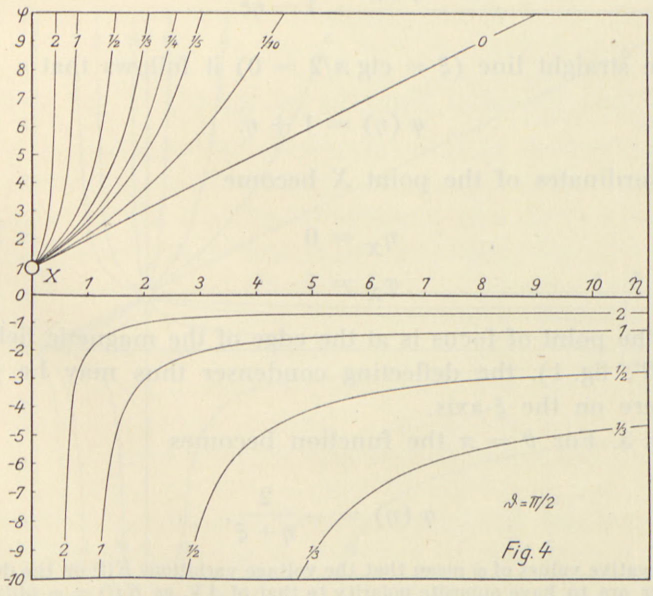
VI. Graphic Representation of the Function $\varphi(\eta)$.

For the illustration in detail of the experimental conditions under which a focusing may be carried out, the function $\varphi(\eta)$, $(\eta > 0)$, is represented graphically with the parameter ξ lying between 0 and $+\infty$ for $\vartheta = \pi/3$, $\pi/2$, π and $3\pi/2$ (figs. 3—6). The general form of the curves, which was discussed in section IV, will be recognized.

Fig. 3. For $\vartheta = \pi/3$ the formula for $\varphi(\eta)$ has the following form:

$$\varphi(\eta) = \frac{0{,}500 + 0{,}866\,\eta}{0{,}866\,(1 - \eta\xi) + 0{,}500\,(\eta + \xi)}. \tag{37}$$





Figs. 3 and 4. Graphical representation of $\varphi(\eta)$. The numbers shown on the curves are values of the parameter ξ. The values of ϑ are $\pi/3$ and $\pi/2$ in the respective figures.

Let us as an example follow the course of $\varphi(\eta)$ for $\xi = 2$. At low values of η, $\varphi(\eta)$ has a low positive value. With increasing η, $\varphi(\eta)$ rises towards the point of singularity at $\eta_\infty = 1{,}514$. This corresponds geometrically to the approach of the point of focus F to F' (fig. 1). If F' is passed, $\varphi(\eta)$ will begin with high negative values, which will soon decrease*. It is evident that the sharpness of the singularity is greatly dependent on ξ.

For $\xi = \operatorname{ctg} \vartheta = 0{,}577$, $\varphi(\eta)$ is a straight line of the form (24), viz.

$$\varphi(\eta) = 0{,}750\,\eta + 0{,}433. \tag{38}$$

The coordinates of the point X (cf. (22)) will be

$$\left.\begin{aligned} \eta_X &= 0{,}577, \\ \varphi_X &= 0{,}866. \end{aligned}\right\} \tag{39}$$

The point Y lies outside the figure since $\eta_Y < 0$.

Fig. 4. For $\vartheta = \pi/2$ we get the simple formula

$$\varphi(\eta) = \frac{1+\eta}{1-\eta\xi}. \tag{40}$$

For the straight line ($\xi = \operatorname{ctg} \pi/2 = 0$) it follows that

$$\varphi(\eta) = 1 + \eta. \tag{41}$$

The coordinates of the point X become

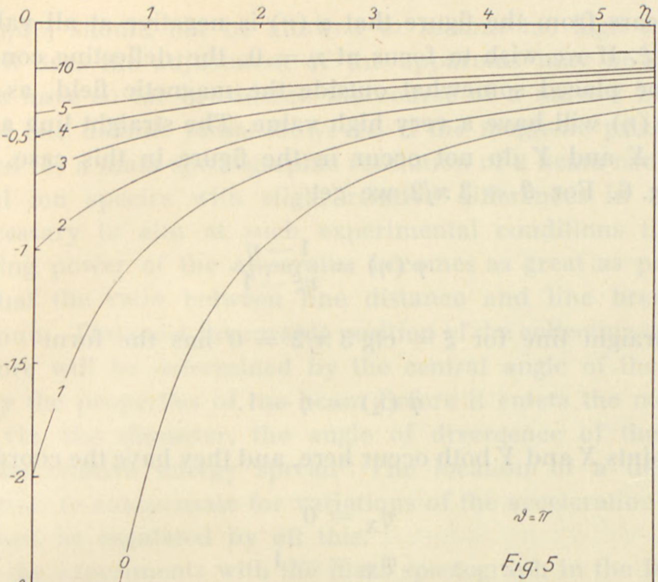
$$\left.\begin{aligned} \eta_X &= 0 \\ \varphi_X &= 1 \end{aligned}\right\} \tag{42}$$

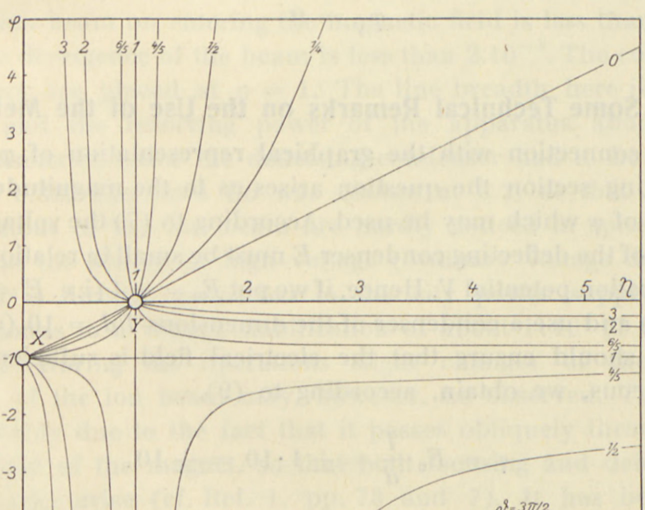
When the point of focus is at the edge of the magnetic field (the point R', fig. 1), the deflecting condenser thus may be placed anywhere on the ξ-axis.

Fig. 5. For $\vartheta = \pi$ the function becomes

$$\varphi(\eta) = -\frac{2}{\eta + \xi}. \tag{43}$$

* Negative values of φ mean that the voltage variations $E(t)$ on the deflecting condenser are to have opposite polarity to that of ΔV, as $E(t) = \varphi \cdot (d/l) \cdot \Delta V(t)$. If the d. c. linear amplifier[3] supplies voltage variations of the same sign as ΔV, it is only necessary to interchange the terminals to the condenser plates a and b (cf. fig. 1).





Figs. 5 and 6. Graphical representation of $\varphi(\eta)$. The parameter ϑ has the values π and $3\pi/2$ in the respective figures.

It appears from the figure that $\varphi(\eta)$ is negative at all values of η and ξ. If we wish to focus at $\eta = 0$, the deflecting condenser must be placed somewhat outside the magnetic field, as otherwise $\varphi(\eta)$ will have a very high value. The straight line and the points X and Y do not occur in the figure in this case.

Fig. 6. For $\vartheta = 3\pi/2$ we get

$$\varphi(\eta) = \frac{1-\eta}{\eta\xi - 1}. \tag{44}$$

The straight line for $\xi = \operatorname{ctg} 3\pi/2 = 0$ has the form

$$\varphi(\eta) = \eta - 1. \tag{45}$$

The points X and Y both occur here, and they have the coordinates

$$\left.\begin{aligned} \eta_X &= 0 \\ \varphi_X &= -1 \end{aligned}\right\} \tag{46}$$

$$\left.\begin{aligned} \eta_Y &= 1 \\ \varphi_Y &= 0 \end{aligned}\right\} \tag{47}$$

VII. Some Technical Remarks on the Use of the Method.

In connection with the graphical representation of φ in the preceding section the question arises as to the magnitude of the values of φ which may be used. According to (7) the voltage variations of the deflecting condenser E must be small in relation to the acceleration potential V. Hence, if we put $E_0 = \pm 1$ (i.e. $E = \pm \Delta V$; cf. (2)) and use a condenser of the dimensions $l/d = 10$ (cf. (7)), which should ensure that the electrical field is sufficiently homogeneous, we obtain, according to (9),

$$\varphi = E_0 \cdot \frac{l}{d} = \pm 1 \cdot 10 = \pm 10. \tag{48}$$

It appears from figs. 3—6 that within this limit of φ, which may even in a given case be exceeded considerably, it will generally be possible to choose a suitable combination of η and ξ, so that a focusing may be effected. It should, however, be taken into consideration that the slope of the curve at the operating point

$d\varphi(\eta)/d\eta$ should not be allowed to assume too high a value, for if it does the adjustment of the apparatus may be critical.

We have so far considered the course of a narrow beam of ions of one definite mass. However, if the magnetic prism is to be used for a mass-spectographic resolution of a beam containing several ion species with slight relative differences in mass it is necessary to aim at such experimental conditions that the resolving power of the apparatus becomes as great as possible, i. e. that the ratio between line distance and line breadth is maximum. The most favourable position of the collecting cylinder therefore will be determined by the central angle of the prism and by the properties of the beam before it enters the magnetic field, viz. the diameter, the angle of divergence of the beam and the relative energy spread[2]. The location of a deflecting condenser to compensate for variations of the acceleration potential must be regulated by all this.

In the experiments with the mass spectograph in the Institute for Theoretical Physics[2] a magnetic field with a central angle of $\vartheta = \pi/2$ and a mean radius of $r = 80$ cm is used. The diameter of the ion beam on entering the magnetic field is less than 1 cm, and the divergence of the beam is less than 2.10^{-3}. The collecting cylinders are placed at $\eta = 1$. The line breadth here is about 3 mm and the resolving power of the apparatus about 300. For reasons of space the deflecting condenser had to be placed at $\xi = 0.38$. The ratio l/d was chosen at 3.2, so that $E_0 \approx 1$. Oscillations of the ion beam are hardly noticed in spite of the fact that the source of high voltage (ordinary voltage doubling circuit) is directly connected to the city power supply. This corresponds to a voltage stabilization of about $1:10^4$ (7 volts at 70.000). During the operations slight changes of the cross section of the ion beam may, however, be observed. They are presumably due to the fact that it passes obliquely through the stray field of the magnet, so that both focusing and defocusing effects may arise (cf. Ref. 1, pp. 73 and 7). It has been observed that these effects have no influence on the mass-dispersion of the apparatus.

A diagram of an arrangement by means of which it is possible in a simple way to produce voltage variations proportional to ΔV has already been published[3]. Perhaps it should be added that the d. c. linear amplifier consists of two stages with a total amplification of about 1,400. In the first stage is used an *AF* 7-valve, while in the second stage a transmitting valve is applied corresponding to the Philips type *PC* 1,5/100, which is fed from a 2,000 volt d. c. tension engine. The d. c. anode tension of the latter valve is 1,000 volts (corresponding to $\Delta V = 0$), which in the present arrangement is compensated for by means of a small rectifier instead of batteries. The voltage variation on the deflecting condenser is a linear function of the input voltage within the region $\pm$ 800 volts. The various parts of the apparatus are well screened in order to avoid disturbances from the outside.

In certain experiments, i. e. those of nuclear physics, it is of great importance that the accelerated ions always strike the target or collecting cylinder with uniform energy. This may be accomplished by insulating these electrodes and supplying them with voltage variations of the magnitude $E' = \Delta V$, since the difference in voltage between the ion source and the electrodes is in this case always V.[3] This method of compensation for the ripple and other fluctuations of the voltage may be useful even without magnetic analysis of the beam. By means of the focusing method proposed here it is, however, possible to make the ion beam pass through a magnetic deflecting field first, and thus to select one definite kind of ions for further experiments. The electrical circuits will become particularly simple if the experimental conditions for focusing are chosen so that $E = \Delta V$ ($E_0 = 1$), as in this case it is possible to connect the deflecting condenser directly to the target.

VIII. Summary.

In the mass-spectrographic analysis of a narrow beam of ions, uniform with respect to energy, by means of a homogeneous wedge-shaped magnetic field even small variations of the accelerating potential of the ions ΔV will have a very disturbing effect, as the ion beam at the collecting cylinder will fluctuate. This movement may be compensated for by making the ion beam, before it enters the magnetic field, pass through a de-

flecting condenser, one plate of which is grounded, while the other is supplied with a voltage proportional to ΔV. The magnitude of the proportionality factor E_0 is mainly determined by the central angle of the wedge-shaped field and by the position of the condenser and the collecting cylinder. The construction of the ion paths by means of geometrical optics shows certain very characteristic features, the discussion of which is closely connected with the general theory of the passage of charged particles through a magnetic prism. To facilitate the use of the method in practice a number of graphic representations are given, from which it is possible to read the magnitude of the proportionality factor E_0 for given experimental conditions. Finally, various technical questions related to the practical application of the method are discussed.

IX. Acknowledgments.

The present paper has been prepared in connection with the experiments made with the mass spectrograph of the Institute for Theoretical Physics. The author wants to offer his best thanks to the director of the Institute, Professor Niels Bohr, for his constant interest in the present investigations. Further I want to offer my thanks to Mr. Aage Bohr, M. Sc., for various discussions on the general theory of the passage of charged particles through magnetic fields, and to Mr. P. Kristensen, B. Sc., for assisting in the mathematical analysis of the function φ. Finally I offer my thanks to Mr. E. Dissing Sørensen, C. E., for his cooperation in the technical construction and testing of the apparatus.

X. References.

1) J. Koch, Diss. Copenhagen (Publ. by Thaning og Appel, 1942).
2) J. Koch and B. Bendt-Nielsen, D. Kgl. Danske Vidensk. Selskab, Mat.-fys. Medd. **21**, 8 (1944).
3) J. Koch, Phys. Rev. **69**, 238 (1946).
4) R. Herzog, Zs. Phys. **89**, 447 (1934).
5) N. F. Barber, Proc. Leeds Phil. Soc. **2**, 427 (1933).
6) W. E. Stephens, Phys. Rev. **45**, 513 (1934).
7) C. Reuterswärd, Arch. f. Mat., Astron. och Fysik **30**, 7 (1944).

Indleveret til Selskabet den 19. Marts 1947.
Færdig fra Trykkeriet den 4. Februar 1948.